and

$$\left(\frac{dk}{dx}\right)^2+\left(\frac{dk}{dy}\right)^2+\left(\frac{dk}{dz}\right)^2+4\frac{dk}{dh}=0.$$

In this case we find $\psi(h)=\left((a^2+h)(b^2+k)(c^2+h)\right)^{\frac{1}{2}}$, and the above general expressions lead to the known results.

II. Supplement to a Paper, read January 27, 1859, "On the Thermodynamic Theory of Steam-engines with dry Saturated Steam, and its application to practice." By W. J. Macquorn Rankine, C.E., F.R.S. &c.*

(Abstract.)

This supplement gives the dimensions, tonnage, indicated horse-power, speed, and consumption of fuel, of the steam-ships whose engines were the subjects of the experiments referred to in the original paper. Results are arrived at respecting the available heat of combustion of the coal employed, and the efficiency of the furnaces and boilers, of which the following is a summary:—

No. of experiment.	Kind of boiler.	Total heat of combustion of 1 lb. of coal in ft.-lbs., estimated from chemical composition.	Available heat of combustion of 1 lb. of coal in ft.-lbs. computed from efficiency of steam and weight of coal burned per I.H.P.	*Available heat*, total heat, = efficiency of furnace and boiler.
I.	Improved Marine Boilers of ordinary proportions.	10,000,000	5,420,000	0·542
III.		10,000,000	5,300,000	0·53
II.	Boiler chiefly composed of small vertical water-tubes, with very great heating surface.	11,560,000	10,110,000	0·88

Available Heat of Combustion of 1 lb. of coal

$$=\frac{1{,}980{,}000 \text{ ft.-lbs.}}{\text{Efficiency of steam} \times \text{lb. coal per I. H. P. per hour}}.$$

* Phil. Trans. 1859, p. 177; and Proceedings of the Royal Society, vol. ix, p. 626.